NATURAL OSCILLATIONS OF GAS FLOWING PAST A LATTICE OF FLAT PLATES

V. B. Kurzin

The splicing method is used to solve the problem of natural oscillations of a gas flowing past a flat plate lattice. In addition, the natural oscillations of a gas in an infinite plane, simulating the natural oscillations of a gas in an annular channel, are examined under the condition of spatial periodicity.

Interest in the study of this question was kindled by the results of several studies of the oscillations of flat plate lattices in subsonic gas flow [1-4]. In these studies it was found that for definite combinations of the lattice and approaching stream parameters the unsteady aerodynamic characteristics of the plates depend markedly on these parameters. With regard to physical interpretation, such phenomena have been explained as acoustical resonance of the gas perturbations caused by vibrations of the profiles with the natural oscillations of the gas in the lattice region in question. In this connection it has been noted that the aerodynamic damping of the lattice vibrations is reduced significantly in the resonant regimes.

In [5] this fact was noted in a study of vibrations in axial compressors. The present paper presents for the first time relations which define the values of the natural frequencies of gas oscillations in an annular channel in the circumferential direction. These same relations, but in a different form and by different methods, have been obtained in [6, 7].

1. We shall first examine the problem of natural oscillations of gas flow in an infinite plane. The periodic solutions of this problem will be a model of the natural oscillations of a gas in an annular channel. The problem reduces mathematically to finding the solution, bounded over the entire plane, of the equation for the amplitude of the unsteady component of the flow velocity potential φ. In the x, y dimensionless Cartesian coordinate system, referred to the characteristic length c, this equation has the form

$$(1-M^2)\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}-2kMj\frac{\partial\varphi}{\partial x}+k^2\varphi=0$$
$$\varphi'=\varphi(x,y)e^{j\omega t},\qquad M=\frac{U}{a},\qquad k=\frac{\omega c}{a} \tag{1.1}$$

Here ω is the frequency of the gas oscillations, U is the undisturbed gas flow velocity along the x axis, and a is the sound speed in the undisturbed flow.

By rotation of the x, y axis through the angle β, we obtain the new dimensionless coordinates ξ, η

$$\xi=x\cos\beta-y\sin\beta,\qquad \eta=x\sin\beta+y\cos\beta \tag{1.2}$$

In this coordinate system (1.1) becomes

$$\Delta\varphi-M^2\cos^2\beta\frac{\partial^2\varphi}{\partial\xi^2}-M^2\sin^2\beta\frac{\partial^2\varphi}{\partial\eta^2}-M^2\sin 2\beta\frac{\partial^2\varphi}{\partial\xi\,\partial\eta}-2kMj\cos\beta\frac{\partial\varphi}{\partial\xi}-2kMj\sin\beta\frac{\partial\varphi}{\partial\eta}+k^2\varphi=0 \tag{1.3}$$

We seek the general solution of (1.3) in the class of periodic functions in the direction η with period L, equal to the length of the circumference of the corresponding annular channel, also refrered to c. Then the function φ is representable by the Fourier series

$$\varphi=\sum_{n=-\infty}^{\infty}f_n(\xi)\exp\frac{j2\pi n\eta}{L} \tag{1.4}$$

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 10, No. 5, pp. 68-75, September-October, 1969. Original article submitted June 2, 1969.

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and the particular periodic solution of (1.3) can be represented in the form

$$\varphi_n = \exp(\lambda_n \xi + j\, 2\pi n \eta / L) \tag{1.5}$$

Substituting (1.5) into (1.3), we obtain the characteristic equation for determining λ_n, whose solution has the form

$$\lambda_n = j\lambda_{1n} \pm \lambda_{2n}, \qquad \lambda_{1n} = \frac{M\cos\beta}{1 - M^2\cos^2\beta}\left[k + \frac{2\pi n}{L} M \sin\beta\right]$$
$$\lambda_{2n} = \frac{1}{1 - M^2\cos^2\beta}\left[\left(\frac{2\pi n}{L}\right)^2 (1 - M^2) - 2k\frac{2\pi n}{L} M \sin\beta - k^2\right]^{1/2} \tag{1.6}$$

Thus the general solution of (1.3), satisfying the periodicity condition in the direction of the η axis, has the form

$$\varphi = \sum_{n=-\infty}^{\infty} \exp\frac{j2\pi n\eta}{L}\,[a_n \exp(j\lambda_{1n} + \lambda_{2n})\,\xi + b_n \exp(j\lambda_{1n} - \lambda_{2n})\,\xi] \tag{1.7}$$

If the radicand in (1.6) for λ_{2n} is less than or equal to zero, the corresponding term of the series (1.7)

$$\varphi_n = \exp[j(\lambda_{1n}\xi + 2\pi n\eta / L)] \cos[\lambda_{2n}'(\xi + \delta)] \quad (\lambda_{2n} = j\lambda_{2n}') \tag{1.8}$$

where δ is an arbitrary number, will be the eigenfunction of the problem in question. It satisfies (1.3) and is bounded over the entire plane. Such solutions of the Helmholtz equation for an infinite plane are presented, for example, in [8].

We isolate from the solution (1.8) the factor $\exp(j\lambda_{1n}\xi)$, characterizing the transport of the gas disturbance by the flow in the ξ direction, setting $\varphi_n = \exp(j\lambda_{1n}\xi)\varphi_n^*$. Then the function $\varphi_n^*\exp(j\omega t)$ will be the superposition of two traveling waves, propagating in directions symmetric with respect to the η axis. In the limiting case, when $\lambda_{2n} = 0$, the function $\varphi_n^*\exp(j\omega t)$ is a traveling wave propagating only in the direction of the η axis.

We introduce the notations

$$L = Nh, \qquad n = n_1 N + m, \qquad \mu = 2\pi m / N$$
$$(m = 0, 1, \ldots, N-1;\ n_1 = 0, \pm 1, \pm 2, \ldots) \tag{1.9}$$

where N is some natural number. Then

$$\frac{2\pi n}{L} = \frac{2\pi n_1 + \mu}{h} \tag{1.10}$$

and we can note that the condition $\lambda_{2n} = 0$ coincides with the condition for acoustic resonance of the natural oscillations of a gas in an infinite plane with disturbances caused by a pulsing dipole chain [7], arranged along the η axis with the spacing h

$$2\pi n_1 + \mu = \frac{kh}{1 - M^2}\,[M\sin\beta \pm \sqrt{1 - M^2\cos^2\beta}] \qquad (n_1 = 0, \pm 1, \ldots) \tag{1.11}$$

From the physical viewpoint this means that the dipole chain, radiating disturbances, resonates with those natural oscillations of the gas which do not contain waves approaching the η axis from infinity on the left or right.

As was noted in [2-5], the condition (1.11) also defines certain characteristics of unsteady gas flow through a planar profile lattice. In this case the parameter h is the dimensionless lattice pitch, referred to the profile semichord c; the parameter β is the lattice stagger angle. Like the pulsing dipole chain, the lattice is a source of disturbance of the gas and for synchronous oscillations of its profiles with the same amplitudes and the constant phase shift μ it excites the corresponding natural oscillation mode in the infinite plane. In this case the interaction of the lattice profiles with the gas decreases sharply, and the aerodynamic damping of the profile oscillations also diminishes. In terming this phenomenon for flow past lattices acoustic resonance, we must note its arbitrariness, since the natural oscillations of the gas in the "lattice" region in the general case do not coincide with the natural oscillations in question.

2. In the case of subsonic gas flow past a lattice of flat plates the eigenvalue problem consists in finding the nontrivial solution of (1.3) with the condition of boundedness of the solution at infinity behind and ahead of the lattice (Fig. 1)

$$\varphi < \infty \quad \text{for} \quad |\xi| \to \infty \tag{2.1}$$

and with the uniform condition of gas nonpenetration through the plates

$$\partial\varphi / \partial y = 0 \quad \text{for} \quad y = nh\cos\beta$$
$$nh\sin\beta < x < nh\sin\beta + 2 \tag{2.2}$$

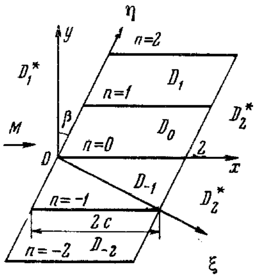

Fig. 1

We consider only those flows in which there are no vortex wakes behind the plates. We shall seek these solutions with the aid of the splicing method [9]. Following this method, we divide the flow region into the regions D_1^* and D_2^* located respectively to the left and right of the lattice and bounded by lines connecting the leading and trailing edges of the plates) and the regions D_n (between the plates (Fig. 1)). In accordance with the periodic function concept (1.7), in the regions D_1^* and D_2^* the most general expression for the eigenfunctions, with account for the substitution (1.11), has the form

$$\varphi = \sum_{m=0}^{N-1} \exp\left(j\mu\frac{\eta}{h}\right) \sum_{n=-\infty}^{\infty} \exp\left(j2\pi n\frac{\eta}{L} + \lambda_{1n}\xi\right)\left[a_{mn_1}e^{\lambda_{2n}\xi} + b_{mn_1}e^{-\lambda_{2n}\xi}\right] \tag{2.3}$$

In the following it will be shown that any unknown function in these regions is described by one of the terms of the sum over m. Moreover, it follows from the condition (2.1), and from the condition of the absence of waves coming from infinity, that the coefficients a_{mn_1} are zero in the region D_2^* and the coefficients b_{mn_1} are zero in the region D_1^*. Therefore we take for region D_1^*

$$\varphi_1^* = \exp\left(j\mu\frac{\eta}{h}\right) \sum_{n=-\infty}^{\infty} a_n \exp\left[(j\lambda_{1n} + \lambda_{2n})\xi + j2\pi n\frac{\eta}{h}\right]; \tag{2.4}$$

for region D_2^*

$$\varphi_2^* = \exp\left(j\mu\frac{\eta}{h}\right) \sum_{n=-\infty}^{\infty} b_n \exp\left[(j\lambda_{1n} - \lambda_{2n})(\xi - 2\cos\beta) + j2\pi n\frac{\eta}{h}\right] \tag{2.5}$$

In (2.4) and (2.5), and also hereafter, the subscript 1 on n_1 in the constants a_{mn_1}, b_{mn_1} is dropped.

The functions φ_1^* and φ_2^* and their first derivatives will be continuous in the regions D_1^* and D_2^*, except for possibly the values $\eta = sh$ and $\eta = 2\sin\beta + sh$ $(s = 0, \pm 1, \pm 2, \ldots)$ on their boundaries corresponding to the coordinates of the profile edges. We know from slender wing theory that at these points the derivative of the velocity potential function may have a singularity of the $(r - r_s)^{-1/2}$ type, where r_s is the coordinate of one of the edges of the s-th profile and r is the radius-vector of the variable coordinate. However, in spite of this singularity the coefficients a_n' and b_n' of the series for the derivatives of the functions φ_1^* and φ_2^*, analogous to the series of (2.4) and (2.5), tend to zero as $n \to \infty$, namely,

$$a_n'|_{n\to\infty} = \frac{2}{h}\int_0^h \left[\frac{c_0}{\sqrt{\eta(h-\eta)}} + f(\eta)\right]\exp\left(j2\pi n\frac{\eta}{h}\right)d\eta = \frac{2\pi c_0}{h} J_0(nh) + o(n^{-1}) \to 0$$

Here the term with the singularity from the derivatives of the functions $\varphi_1^*(0, \eta)$, $\varphi_2^*(2\cos\beta, \eta)$ is separated in the form $c_0/\sqrt{\eta(h-\eta)}$. Consequently, the series for the first derivatives of the velocity potential functions converge for any value of η except for the coordinates of the profile edges.

In the regions D_n the general expression for the eigenfunctions will be defined by the solution of mixed problems of the form

$$\varphi_n = \varphi_1^* \quad \text{for} \quad \xi = 0, \qquad \varphi_n = \varphi_2^* \quad \text{for} \quad \xi = 2\cos\beta$$
$$\partial\varphi_n / \partial y = 0 \quad \text{for} \quad y = nh\cos\beta, \; y = (n+1)h\cos\beta \tag{2.6}$$

We shall seek the functions φ_n in the form of an infinite series of solutions of the equation (1.1), each of which satisfies the nonpenetration condition. Considering that the first two conditions (2.6) for the different regions differ only in the factor $\exp(jn\mu)$, the general expression for the function φ_n can be written as follows:

$$\varphi_n = e^{j(\sigma x + n\mu)} \sum_{m=0}^{\infty} \left[c_m e^{\lambda_m(x-2)} + d_m e^{-\lambda_m x}\right] \cos\left[\pi m\left(\frac{y}{h\cos\beta} - n\right)\right]$$
$$\lambda_m = \frac{1}{1-M^2}\left[\left(\frac{\pi m}{h\cos\beta}\right)^2(1-M^2) - k^2\right]^{1/2}, \qquad \sigma = \frac{kM}{1-M^2} \tag{2.7}$$

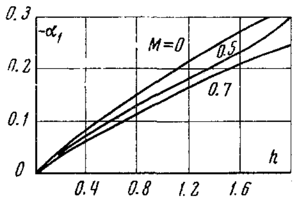

Fig. 2

We shall determine the constants a_n, b_n of the functions (2.4) and (2.5) and also the constants c_m and d_m of the function (2.7) in accordance with the splicing method from the condition of continuity of the unknown function and its normal derivative on the lines $\xi = 0$ and $\xi = 2 \cos \beta$, except possibly the points corresponding to the coordinates of the profile edges. In so doing the first two conditions (2.6) for the function (2.7) are automatically satisfied.

We note that it suffices to splice the functions (2.4) and (2.5) with the function (2.7) only in the interval of a single step h. On the remaining segments of the lines $\xi = 0$ and $\xi = 2 \cos \beta$ splicing is accomplished by virtue of the periodicity condition. Since the expressions (2.4) and (2.5) in the interval of a single step describe an arbitrary function of the sought solution, we can state that summation over the index m in (2.3) does not yield a more general representation of the sought solution and can be dropped.

Thus, equating the functions (2.4) and (2.5) and their derivatives in the ξ direction to the function (2.7) and its derivative on the lines $\xi = 0$ and $\xi = 2 \cos \beta$ respectively, we obtain four relations which connect the unknown constants.

For the sake of brevity we write out only the two of these relations which satisfy the continuity conditions on the line $\xi = 0$:

$$\exp\frac{j\mu\eta}{h}\sum_{n=-\infty}^{\infty} a_n \exp\frac{j2\pi n\eta}{h} = \sum_{m=0}^{\infty}\left[c_m e^{\lambda_m(\eta\sin\beta-2)} + d_m e^{-\lambda_m\eta\sin\beta}\right] e^{j\sigma\eta\sin\beta}\cos\frac{\pi m\eta}{h}$$

$$\exp\frac{j\mu\eta}{h}\sum_{n=-\infty}^{\infty} a_n (j\lambda_{1n}+\lambda_{2n})\exp\frac{j2\pi n\eta}{h} = \sum_{m=0}^{\infty} e^{j\sigma\eta\sin\beta}\Big\{\cos\beta\left[c_m(\lambda_m+j\sigma)e^{\lambda_m(\eta\sin\beta-2)} + d_m(-\lambda_m+j\sigma)e^{-\lambda_m\eta\sin\beta}\right] \tag{2.8}$$

$$\times\cos\frac{\pi m\eta}{h} + \frac{\pi m}{h}\operatorname{tg}\beta\left[c_m e^{\lambda_m(\eta\sin\beta-2)} + d_m e^{-\lambda_m\eta\sin\beta}\right]\sin\frac{\pi m\eta}{h}\Big\}$$

To find the unknown constants, we transfer from the system (2.8) to an infinite system of algebraic equations. To do this we multiply these relations on the left and right by the function $\exp[-j(2\pi n + \mu) \times \eta/h]$ $(n = 0, 1, 2, \ldots)$ and integrate over η from 0 to h. Then we have

$$a_n = \sum_{m=0}^{\infty}\left[c_m e^{-2\lambda_m}A_{nm} + d_m B_{nm}\right]$$

$$(j\lambda_{1n}+\lambda_{2n})\,a_n = \sum_{m=0}^{\infty}\left\{c_m e^{-2\lambda_m}\left[(\lambda_m+j\sigma)\cos\beta - \frac{(\pi m)^2\operatorname{tg}\beta}{h\theta_{nm}'}\right]A_{nm} + d_m\left[(-\lambda_m+j\sigma)\cos\beta - \frac{(\pi m)^2\operatorname{tg}\beta}{h\theta_{nm}''}\right]B_{nm}\right\} \tag{2.9}$$

Here

$$A_{nm} = \frac{\theta_{nm}'}{\theta_{nm}'^2+(\pi m)^2}\left[(-1)^m\exp(\theta_{nm}')-1\right], \quad B_{nm} = \frac{\theta_{nm}''}{\theta_{nm}''^2+(\pi m)^2}\left[(-1)^m\exp(\theta_{nm}'')-1\right]$$

$$\theta_{nm}' = j(\sigma h\sin\beta - 2\pi n - \mu) + \lambda_m h\sin\beta, \quad \theta_{nm}'' = j(\sigma h\sin\beta - 2\pi n - \mu) - \lambda_m h\sin\beta$$

The constants a_n are easily excluded from each pair of the system (2.9). Performing similar operations for the relations satisfying the continuity conditions on the line $\xi = 2\cos\beta$, we obtain the system of equations for determining the unknown constants c_m and d_m in the form

$$\begin{gathered}\sum_{m=0}^{\infty}\left\{c_m\left[\cos\beta(\lambda_m+j\sigma) - \frac{(\pi m)^2\operatorname{tg}\beta}{h\theta_{nm}'} - (j\lambda_{1n}+\lambda_{2n})\right]A_{nm}e^{-2\lambda_m}\right.\\ \left.+\,d_m\left[\cos\beta(-\lambda_m+j\sigma) - \frac{(\pi m)^2\operatorname{tg}\beta}{h\theta_{nm}''} - (j\lambda_{1n}+\lambda_{2n})\right]B_{nm}\right\} = 0\\ \sum_{m=0}^{\infty}\left\{c_m\left[\cos\beta(\lambda_m+j\sigma) - \frac{(\pi m)^2\operatorname{tg}\beta}{h\theta_{nm}'} - (j\lambda_{1n}-\lambda_{2n})\right]A_{nm}\right.\\ \left.+\,d_m\left[\cos\beta(-\lambda_m+j\sigma) - \frac{(\pi m)^2\operatorname{tg}\beta}{h\theta_{nm}''} - (j\lambda_{1n}-\lambda_{2n})\right]B_{nm}e^{-2\lambda_m}\right\} = 0\\ (n = 0, \pm 1, \pm 2, \ldots)\end{gathered} \tag{2.10}$$

Since (2.10) will be homogeneous, existence of its nontrivial solution is possible only if the determinant composed of the coefficients of the unknown constants equals zero. Thus, the problem in question is reduced to the determination of the eigenvalues of the infinite system of algebraic equations (2.10).

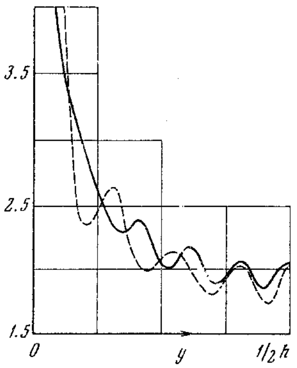

Fig. 3

We note that the approximate values of the eigenvalues of (2.10) can be found with prespecified accuracy from the truncated system, i.e., for its solution by the method of reduction. However, even this problem presents considerable computational difficulty in the general case. As an illustration we analyze the natural oscillations of the gas in a lattice region using the very simple example of flow through an unstaggered lattice.

3. In the case of an unstaggered lattice ($\beta = 0$) the infinite system of algebraic equations simplifies considerably. In fact, for $\beta = 0$ the coefficients $A_{nm} = B_{nm}$, $\theta_{nm}' = \theta_{nm}''$, $\sigma = \lambda_{1n}$, and the system of equations (2.10) takes the form

$$\sum_{m=0}^{\infty} [c_m(\lambda_m - \lambda_{2n})\, e^{-2\lambda_m} - d_m(\lambda_m + \lambda_{2n})]\, A_{nm} = 0,$$

$$\sum_{m=0}^{m} [c_m(\lambda_m + \lambda_{2n}) - d_m(\lambda_m - \lambda_{2n})\, e^{-2\lambda_m}]\, A_{nm} = 0$$

$$(n = 0, \pm 1, \pm 2 \ldots)$$

Hence $c_m = d_m$ and (2.10) thereby becomes

$$\sum_{m=0}^{\infty} c_m [\lambda_m + \lambda_{2n} - e^{-2\lambda_m}(\lambda_m - \lambda_{2n})]\, A_{nm} = 0 \qquad (n = 0, 1, 2 \ldots) \tag{3.1}$$

We first of all note that for $\mu = 0$ the coefficients $A_{nm} = 0$ if $m \neq 2n$. But since $\lambda_m = \lambda_{2n}$ for $m = 2n$, it follows from (3.1) that all the constants c_m and d_m, and therefore a_n and b_n as well, are zero in this case. Thus, for $\mu = 0$ there is no nontrivial solution of (3.1). It is further not difficult to note that if the quantity λ_m vanishes for some fixed value of m two columns of the system determinant coincide.

Thus, the condition $\lambda_m = 0$ defines the combination of parameters

$$k = \frac{\pi m}{h} \sqrt{1 - M^2} \qquad (m = 1, 2, \ldots) \tag{3.2}$$

for which the gas flowing over the unstaggered lattice can perform natural oscillations. The eigenfunctions of (1.1) corresponding to these oscillations have the form

$$\varphi = \exp(j\sigma x) \cos(m\pi y / h) \tag{3.3}$$

We note that (3.2) coincides with (1.11) for $\beta = 0$ and $\mu = \pi$, i.e., in this particular case the natural oscillations of the gas in the lattice region coincide with the natural oscillations of the gas in an infinite plane.

In this case of forced vibrations of a lattice with a frequency satisfying the condition (3.2), the vibration period is a multiple of the time after which a disturbance wave from some point of the profile reaches the corresponding point of the neighboring profile and after reflection returns to the original point. If in this case the vibrations of the neighboring profiles are performed in phase opposition, then the disturbances caused by each of the profiles combine and acoustic resonance will occur.

However the natural oscillations of the gas which arise under the condition (3.2) do not cover the entire spectrum of natural oscillations of practical interest for the case of an unstaggered lattice. The natural oscillations examined include only oscillations in the transverse direction (in the direction of the lattice front). In accordance with the familiar acoustical results for open resonators [10], we can also expect natural oscillations of the gas in the longitudinal direction, since the interprofile channels are essentially just such resonators.

In the first approximation the natural frequencies of such oscillations will be determined from the condition [6]

$$k = (1 - M^2)\, \pi m / 2 \qquad (m = 1, 2, \ldots) \tag{3.4}$$

corresponding to the case of complete reflection of a small-perturbation plane wave from the open ends.

In reality the plane wave is not completely reflected from the open end, it rather interacts with the surrounding space, including the neighboring interprofile channels. Therefore the corresponding eigenfunction will not be a simple plane wave, localized in a single channel, but rather some complex function accounting for this interaction over the entire flow plane. Here the natural frequencies of the oscillations will differ from the values defined by (3.4).

We introduce the parameter α_m, accounting for the correction for the open end in (3.4), so that the reduced natural frequency of the longitudinal oscillations of the gas is

$$k_m = (1 - M^2)\,\pi m\,(1 + \alpha_m)/2 \qquad (m = 1, 2, \ldots) \tag{3.5}$$

The values of the parameter α_m and also the eigenfunction of the sought oscillations can be found with the aid of the solution of (3.1).

Let us examine the example of the calculation for m = 1, $\mu = \pi$. In this case the expressions for the eigenfunctions (2.4), (2.5), (2.7) become

$$\varphi_1^* = \sum_{n=0}^{\infty} a_n \exp\left[(j\sigma + \lambda_{2n})x\right] \sin\frac{(2n+1)\pi y}{h}, \quad \varphi_2^* = \sum_{n=0}^{\infty} b_n \exp\left[(j\sigma - \lambda_{2n})(x-2)\right] \sin\frac{(2n+1)\pi y}{h}$$

$$\varphi_n = \sum_{m=1}^{\infty} e^{j\sigma x}\left[c_m \exp\lambda_m(x-2) + d_m \exp(-\lambda_m x)\right] \cos\frac{2\pi m y}{h}$$

$$\lambda_m = \frac{1}{1-M^2}\left[\left(\frac{2\pi m}{h}\right)^2 (1-M^2) - k^2\right]^{1/2}, \quad \lambda_{2n} = \frac{1}{1-M^2}\left[\left(\frac{2n+1}{h}\right)^2 \pi^2 (1-M^2) - k^2\right]^{1/2}$$

Following the reduction method, (3.1) was truncated for the calculation to N = 30 equations. The vanishing of the determinant of the truncated system for fixed values of M and h was examined as the approximate condition for finding the parameter α_1.

The results of the calculation of the parameter α_1 as a function of the dimensionless spacing h for M = 0, 0.5 and 0.7 are shown in Fig. 2. Analyzing this relation, it is interesting to note that $\alpha_1 \to 0$ as $h \to 0$. From the physical viewpoint this result can be explained by the fact that with increase of the channel length the fraction of the disturbed gas kinetic energy radiated from the open end decreases relative to the energy of the oscillating gas within the channel and approaches zero in the limit for infinite channel length.

We present the calculated values of the first ten unknown coefficients a_n and c_n, normed with respect to a_0 for h = 1 and M = 0.7.

$n =$		0	1	2	3	4
$a_n =$		1.0	0.1660	0.0761	0.0462	0.0320
$-c_n =$	—0.69—	j 3.12	0.2480	0.0898	0.0489	0.0316
$n =$	5	6	7	8	9	10
$a_n =$	0.0241	0.0191	0.0157	0.0132	0.0114	0.0101
$c_n =$	0.0224	0.0168	0.0132	0.0107	0.0089	0.0075

These values agree to within the third place with the corresponding coefficients calculated from the system truncated to 20 equations, which indicates good convergence of the reduction method in this case. However, judging by the decrease of the coefficients the convergence of the derivatives of the unknown functions is poor.

This fact is illustrated by Fig. 3, which shows curves of the normal derivatives of the functions φ_0 (continuous curve) and φ_1^* (dashed) versus y on the line x = 0. We see from these curves that the normal derivatives of the unknown functions to the left and right of the splicing line differ from one another by a magnitude of the order of the error of their approximation by finite trigonometric series. (Splicing of the functions φ_0 and φ_1^* themselves in the case in question is accomplished to within 3-4 significant figures.) In accordance with the order of decrease of the coefficients a_n and b_n, there is a marked singularity of the derivatives of the unknown functions at the edges of the plate. Here these singularities appear at both ends of the plates on the basis of the construction of the solution.

We note that apparently such solutions are physically realizable only for M = 0. In the flow with $M \neq 0$ the solutions in the class with bounded derivatives at the trailing edges of the plate are of greatest practical interest. In the general case these solutions must be sought with account for the vortex wakes, and the corresponding eigenvalues are complex numbers.

The author wishes to thank G. Yu. Stepanov and R. A. Shipov for valuable comments in discussions of the study.

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